Laboratoire d'Economie Forestière

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Abstract

Using the Markowitz mean-value (M-V) portfolio model, we study forest planning looking at arbitration between productivity and risk. By weighting the forest productivity with factors of future climate change effects, we compute the optimal tree species mixes, within reach of forest managers, in ninety French administrative departments. Considering three productivity measures (wood production, carbon sequestration and economic valorization) and their respective variances, we found that: a) optimizing productivity and carbon sequestration yields allocations close to the empirical ones; b) forest managers prefer low variance to high productivity, i.e. their revealed risk aversion is high; and c) unlike maximizing wood productivity or carbon sequestration, which lead to similar portfolios, maximizing the economic value of wood production increases (decreases) wood production and carbon sequestration under risk aversion (neutrality). Under high risk aversion, the economic valorization would lead to a high species specialization, which is very unlikely in reality. In all considered scenarios, the objectives set out in the Kyoto Protocol would be attained, which puts into question its relevance in terms of additionality.

Keywords: bioeconomics, forest planning, mean-variance model, mixed-species forests, climate Change.

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Contents

1 Introduction

Due to climate variations, as well as biotic and abiotic disturbances, the services provided by forest ecosystems are characterized by their strong fluctuations. Furthermore, climate change is expected to alter the provision of these services in a way that is far from being fully understood (Millar et al., 2007).

On one side, the increase of the CO_2 atmospheric concentration may lead to the *carbon fertilization effect*, according to which the growth rate of tree species should increase (Soulé & Knapp, 2006; Knapp et al., 2001). On the other side, climate change may accentuate the risk of tree mortality (Allen et al., 2010; Lindner et al., 2010; Dale et al., 2000).

The objective we have set is to describe a methodology that, selecting a particular mix of tree species, could help to shape the forest ecosystems such that the provision of services is both maximized and resilient to external shocks. For instance, the optimal mix of tree species could lower the risk of seeing the level of forest services deteriorated in the face of climate change.

As regards the forest management and following the results obtained in our calibration on the French forest owners at the departemental level, we consider the preferences of forest managers lie within a continuum between risk aversion and risk neutrality. Put differently, when forest resources are treated as investments that could generate a level of expected utility, their managers would not invest in a combination of tree species – a silvicultural portfolio – if a more favorable portfolio, with different expected return and risk, is achievable. In that sense, the forest manager is considered to be rational, for he or she will be looking for a portfolio that generates the greatest expected utility (Kumar et al., 2014).

Through the mean-variance (M-V) model, the trade-off between the expected return of a portfolio of assets and its combined variance has initially been discussed by Markowitz (Markowitz, 1952). A model then extensively applied, as an arbitrage tool, to numerous economic sectors, forestry included (Pasalodos-Tato et al., 2013).

In such a model, a specific weighted combination of assets, such as tree species, is selected in order to minimize the portfolio variance subjected to a given target return or, equivalently, so as to maximize the expected return given an acceptable level of variance.

When applied to forestry, the portfolio analysis has been employed from the point of view of the forest managers acting as investors, where investments in timberland were compared to other types of investments (e.g. stocks or bonds) in order to maximize the portfolio financial return (Thomson, 1991; Wan et al., 2015). Alternatively, the M-V model has been employed as a decision aid tool to deal with risk and uncertainty, with a portfolio of tree species covered either at the stand level (Knoke et al., 2008; Knoke, 2008; Roessiger et al., 2011), the management level (Knoke et al., 2005; Neuner et al., 2013), or at the regional level (Brunette et al., 2014).

Most of the studies aforementioned are based on Historical Distribution Analysis (HDA), using Monte Carlo simulations. Contrariwise, this paper follows the work by Brunette et al. (2014) and uses a Historical Burn Analysis (HBA): on the basis of the historical data issued from the French National Forest Inventory (IGN), we build a portfolio dependent on the productivities of tree species and their variances, the latter reflecting the production risk. While our model considers three objectives that can be assigned to forest ecosystems (Wood Production – WP, Carbon Sequestration – CS, Economic Value – EV), the optimization has been conducted using species and department specific historical observations of tree growth.

The literature in forest ecology usually states that a low growth level indicates a high mortality risk, for it reflects the tree vigor and is indicative of its survival likelihood (Buchman et al., 1983; Bigler et al., 2004; Dobbertin, 2005). Moreover, many recent works suggest that a high variance in tree growth reflects a high risk of mortality (Ogle et al., 2000; Suarez et al., 2004; McDowell et al., 2010; Heres et al., 2012). Thereby, the environmental stress produces an exaggerated variation of tree-rings, such that greater sensitivity to stress comes down to greater mortality (Hogg et al., 2005; Linares & Camarero, 2012). The amplitude of variation of productivity is thus considered to be a measure of risk ((Tilman et al., 1997; Andreu et al., 2007; Slimani et al., 2014)).

This paper builds on the study by Brunette et al. (2014) and extends the Markowitz portfolio selection, within the silvicultural framework: (a) for different levels of risk aversion exhibited by forest managers; (b) for different climate change scenarios during the optimal allocation; (c) to different maximization objectives, such that WP is compared with CS and EV.

As the portfolio expected output is computed from the historical data, the implicit assumption is that the expected productivity of species would be equivalent to the ones currently observed. However, this invariability assumption is mitigated by the fact that the portfolio simulations are conducted at a relatively small scale, that is, the French administrative departments.

Through simulations, our model yields the following results: a) optimizing productivity and carbon sequestration yields allocations close to the empirical ones; b) forest managers prefer low variance to high productivity, i.e. their revealed risk aversion is high; and c) unlike maximizing wood productivity or carbon sequestration, which lead to similar portfolios, maximizing the economic value of wood production increases (decreases) wood production and carbon sequestration under risk aversion (neutrality). Under high risk aversion, considering the economic value, rather than the wood productivity, would lead to a high specialization in tree species. This is neither likely nor desirable due to the risk which would result from low diversification, not to mention the change of scenery. Considering either scenario, the objectives set out in the Kyoto Protocol would be attained.

After this starting section, the methodology we have used is presented in Section 2. Section 3 is devoted to illustrating simulation examples. Section 4 discusses the results.

2 Methodology

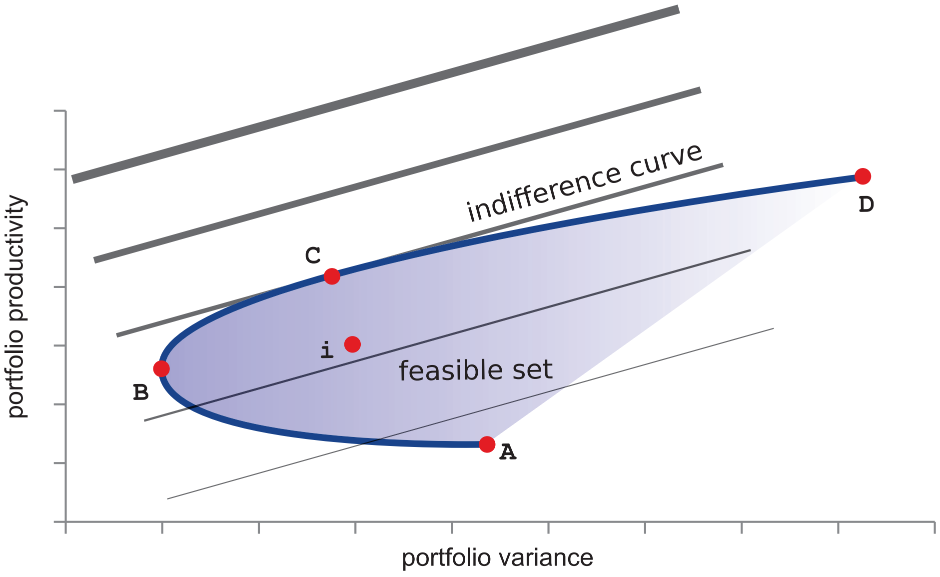
The portfolio allocation can be graphically portrayed as in Fig. 1, where the feasible set of variance-productivity combinations (such as point `i`) in enclosed by the blue curve, and the upper segment of the parabola (`B-D` segment) represents the *efficient frontier* (EF), that is, all the optimal allocations achievable by the decision maker. Thereby, no risk can be lowered at the expense of the productivity level and no productivity can be enhanced without increasing the risk.

In order to simplify the computations, forest managers are assumed to have linear preferences toward risk, in such a way that they trade off variance with productivity on a proportional basis. In this case, the indifference curves can be drawn like a bundle of straight lines, which equation is defined as $productivity = \alpha \times variance + \beta$, where α is the linear risk aversion coefficient, and *productivity* and *variance* refer to the overall expected productivity and variance of the portfolio.

Point `B` ($\alpha \to \infty$) represents the point at which the portfolio variance is the lowest. Agents with α-level of risk aversion are expected to choose the point `C`, at which the tangent indifference curve intersects the efficient frontier. The intercepts between linear curves and the parabola embody the utility of the certainty equivalent. Mathematically, it boils down to solving the following quadratic problem:

$$\begin{array}{rrcll} \max_{x_i,\beta} & Y &=& \beta & \\ s.t. & & & & \\ & x_i &\geqslant& 0 & \forall i \\ & \sum_i x_i &=& 1 & \\ & \sum_i x_i p_i &=& \alpha \sum_i \sum_j x_i x_j \sigma_{i,j} + \beta & \end{array} \qquad (1)$$

Figure 1: Graphical representation of the portfolio allocation



By substitution, the former becomes:

$$\begin{array}{rrll} \min_{x_i} & Y & = \alpha \sum_i \sum_j x_i x_j \sigma_{i,j} - \sum_i x_i p_i & \\ s.t. & & & \\ & x_i & \geqslant 0 & \forall i \\ & \sum_i x_i & = 1 & \end{array} \tag{2}$$

where x_i is the share of asset i, p_i is its productivity and $\sigma_{i,j}$ is the covariance between assets i and j. In this way, $\sum_i x_i p_i$ is the overall portfolio productivity and $\sum_i \sum_j x_i x_j \sigma_{i,j}$ its corresponding variance.

Finally, point `D` (where $\alpha = 0$) is the highest portfolio productivity attainable by the decision maker. Despite its performance, it is more a degenerated solution where only the most productive species remain.

Proposition 1 *With linear utility functions, the portfolio allocation problem is a strictly convex minimization problem.*

Proof. Equation 2 is a sum of linear functions and quadratic terms. Provided that the quadratic terms, which only arise when $i = j$ and $\sigma_{i,j} \equiv \sigma_i^2$, are always positive, they are strictly convex and so is the function (Chiang & Kevin, 2005). ■

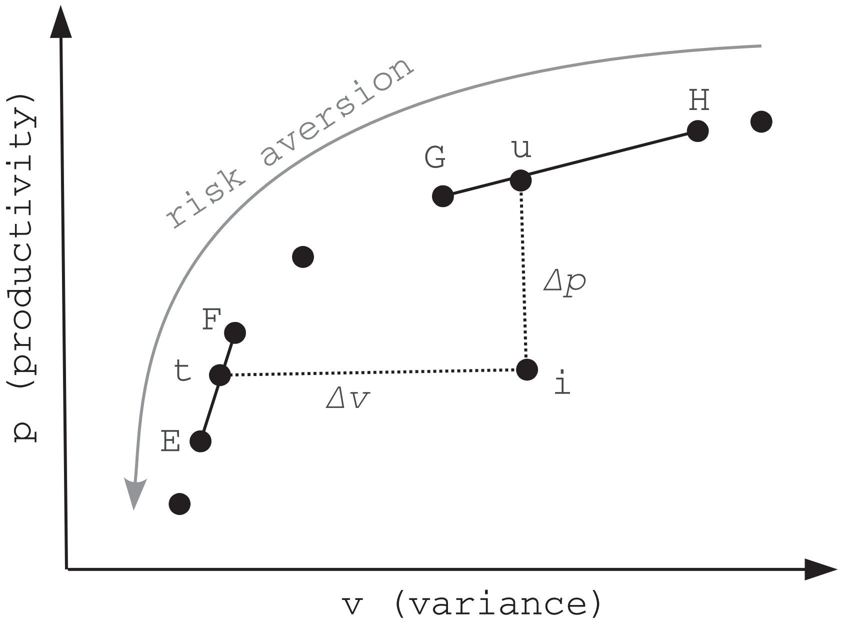
The bounds being linear, we employ QuadProg++ (Di Gaspero, 2007) in order to numerically solve the problem in 2. By means of an active-set dual method, the former is a library which implements the algorithm of Goldfarb & Idnani (1983) for the (convex) quadratic programming problems.

2.1 Empirical risk aversion

Through the use of the M-V model, we measure the distance between the current allocation of tree species and an optimized portfolio. As the empirical point `i` is contained in the space bounded by the efficient frontier, we aim at revealing its corresponding risk aversion coefficient

(α_i) with the purpose of reaching point `C` (Fig. 1). Indeed, the latter belongs to the efficient frontier and displays necessarily the same α_i. To do so, we use a simple linear interpolation as sketched in Fig. 2.

Figure 2: Representation of the interpolation to retrieve the risk aversion



After founding α_t and α_u, we weighted the coefficients using distances $\triangle v$ and $\triangle p$:

$$\begin{aligned}
\alpha_t &= \tfrac{\alpha_F-\alpha_E}{p_F-p_E} \times (p_i - p_E) + \alpha_E \\
\alpha_u &= \tfrac{\alpha_H-\alpha_G}{v_H-v_G} \times (v_i - v_G) + \alpha_G \\
\triangle v &= v_i - \left(\tfrac{(v_F-v_E)(p_i-p_E)}{(p_F-p_E)} + v_E\right) \\
\triangle p &= \left(\tfrac{(p_H-p_G)(v_H-v_G)}{(v_H-v_G)} + p_G\right) - p_i \\
\alpha_i &= \alpha_t \times \tfrac{\triangle p}{\triangle v+\triangle p} + \alpha_u \times \tfrac{\triangle v}{\triangle v+\triangle p}
\end{aligned} \tag{3}$$

We fall on a risk aversion coefficient equal to 70.52 $m^3\ ha^{-1}\ y^{-1}$ for France[1]. This value has been added to the pre-established list of coefficients, so that we could re-run the simulations and find the optimal portfolio at point `C`. The results relative to the intermediate risk aversion in Table 2 and Table 3 refer to such a value.

2.2 Productivity data

The data used on eleven tree species present in France comprises the 1978-2009 time length. The database, coming from the French National Forest Inventory (IFN[2]), included the volume growth, as well as the area occupied per species, per department and per year.

The data happens to be relatively sparse for two reasons. First, individual tree species are often present only in a subset of departments. Second, between 1978 and 2006, the annual

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[1] The reason why the risk aversion coefficient is not adimensional is given in section 2.4.

[2] In 2012, the French National Geographic Institute and the French National Forest Inventory have merged into the French National Institute of Geographic and Forest Information (IGN)

inventory concerned few departments, with a time gap of 10 to 12 years between two inventories in the same department. In 2004, the sampling method changed (IFN, 2004), such that all departments could be simultaneously inventoried. The method has become operative in 2007.

Table 1 shows the number of departments in which the species is present and the number of years during which it has been identified. For example, *Abies alba* Mill appeared 5 times in 35 departments.

Table 1: Presence of forest species in inventories at the department level

	Number of years with observations					
	1	2	3	4	5	Total
Abies alba Mill.	2	3	16	1	35	57
Fagus sylvatica L.		2	11	1	64	78
Larix decidua Mill.	4	2	16		5	27
Picea abies L.	1	5	6	2	47	61
Pinus pinaster Aiton	1	4	8	1	29	43
Pinus sylvestris L.	1	4	13	3	58	79
Pseudotsuga menziesii Franco	3	1	23	1	43	71
Quercus ilex L.	1		5	1	12	19
Quercus petraea Liebl.		1	7	3	73	84
Quercus pubescens Willd.	1	4	22	1	30	58
Quercus robur L.		2	4	1	74	81
Total	14	28	131	15	470	658

In order to build the covariance matrix from eq. 2, we have decided to consider species for which we had a minimum of 4 observations and used a simple linear interpolation to fill the data gaps.

2.3 Climate change multipliers

Climate change multipliers describe the effects of a climate scenario on the variation of the average growth of tree species. By means of a statistical procedure described below, they have been computed for 7 different species[3] by J.D Bontemps and P. Mérian of the Laboratory for the Study of Forest-Wood Resources (LERFoB)[4].

The empirical growth rates – from the IGN data on radial growth obtained by core drilling of the stems – have been correlated to both edaphic and climatic data – the SAFRAN data over the period 1958-2010 – on a high-resolution scale (8 km resolution grid) using a generalized additive model (GAM). The growth rates have then been projected using the CERFACS future climate scenarios.[5] The projections covered the years ranging from 2000 to 2100 for the IPCC[6] scenarios a1b, a2 and b2, which are issued from the ARPEGE-Climate model. The projected growth rates have finally been downscaled at a regional level resolution and converted to multipliers.

2.4 Three objectives

While Brunette et al. (2014) report on optimal portfolios considering the sole wood production (WP), our goal is to extend their analysis to carbon sequestration (CS) and economic valorization (EV). In order to achieve this, the original wood productivities have been multiplied by a species

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[3] When a multiplier of a species in Table 3 was not available, we employed the existing multiplier of the species with the closest ecological needs.

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[5] Centre Européen de Recherche et de Formation Avancée en Calcul Scientifique

[6] Intergovernmental Panel on Climate Change

specific coefficient that converts them into the required measures, which enabled us to recompute the covariance matrices in the new unit measures *i.e.*: *euros* and CO_2 equivalent per hectares.

2.4.1 Carbon sequestration

While undoubtedly a temporary solution, carbon sequestration in carbon pools with long-term turnover (e.g. forests) is a relatively cheap and quick form of Carbon Dioxide Removal Method. We know that the latter can help reducing the cumulative impact of higher temperature (Ciais & Sabine, 2013; Smith & Bustamante, 2014). We thus introduced a portfolio objective in which the optimal allocation would maximize the carbon sequestered in the forest stands. We multiplied the wood productivity by a CO_2 factor $F_s^{\mathrm{CO}_2}$ for each species s:

$$F_s^{\mathrm{CO}_2} = wd_s \times cc_{gs} \times expf_s^b \times expf_s^r \times \frac{44}{12} \tag{4}$$

where wd_s is the wood density by species defined over the oven dry mass over the fresh volume (Chave et al., 2009; Zanne et al., 2009), cc_{gs} is the carbon content by group of species gs (hardwood/softwood) (Lamlom & Savidge, 2003) and $expf_s^b \times expf_s^r$ are respectively the branch and roots expansion factors (Loustau, 2004).

The output is a sequestration productivity per hectare and per year. We find an average value of 55.4 million of tons of CO_2 sequestrated each year by the French metropolitan forests. This volume is comparable to the French National Forestry Office figure [7] obtained when using the method of (Loustau, 2004) and Dupouey & Pignard (2001). We refer to the CO_2 equivalent throughout this paper, to fit the standard terminology of international negociations framework on greenhouses gases, even if only CO_2 is considered in our case.

2.4.2 Economic value

In a similar way, we optimized the forest portfolio with the economic valorization objective, that is, the productivity multiplied by the roadside prices of the corresponding wood.

Such as depicted in Fig. 3, which shows the evolution of prices during the study period, there is a large heterogeneity in the absolute values and variances. From what we observe, the lower the price, the lower the variance.

The price data was collected by the French newspaper *La forêt privée*,[8] diffused twice a month, from 1958 onwards. A detailed description can be found in (Chevalier et al., 2011). We used the annual mean of maximum and minimum prices, issued from the bids recorded during the auction sales in the considered French departments. We matched the tree species and the aforementionned available prices. Only the prices of the highest wood quality have been taken into account.

3 Simulations

Given the productivity data detailed in Section 2.2, and the multipliers explained in Section 2.4, we implemented a simple computer program so as to find the allocation of tree species which maximizes wood production at a minimum variance.

The efficient frontier was built upon a set of 14 risk aversion coefficients, ranging from 0 (point `B` in Fig. 1) to 10,000. We then optimized the portfolio problem for point `D` by selecting the species which best perform as to productivity. In order to account for 90 administrative

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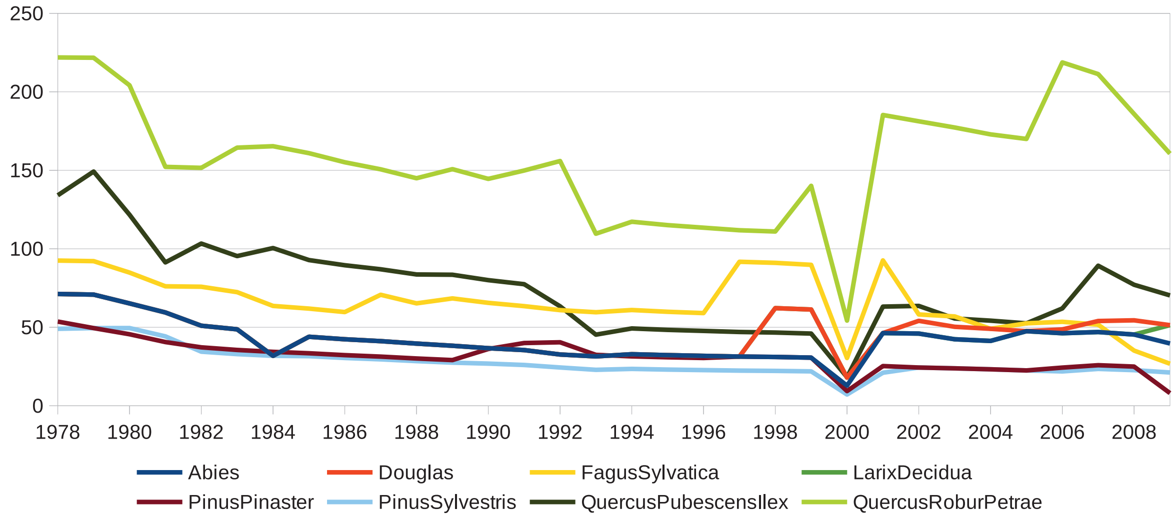
[7] Cf.:French National Forestry Office, key figures (in French)

[8] http://www.laforetprivee.com

Figure 3: Evolution of main wood prices per cubic meter and per species in France



departments, 14 risk aversion coefficients, 3 climate change scenarios, 3 different objectives and 10 time spots, a total of 117,180 simulations had to be run.

The computational constraint of maintaining the optimization problem in its quadratic form prevented us from using the standard deviation or a more elaborated measure of risk, such as the value at risk (VaR) or the conditional value at risk (CVaR) (Wan et al., 2015). In particular, using variance as a risk measure implies that α is defined over the metric used to measure productivity. In order to maintain the meaning of α consistent, we normalized different measures of productivity from Section 2.4 before running the allocation problem.

While the expected productivity of the forest species has been computed from 2010 to 2100, in ten-year steps, using the climate change multipliers described in Section 2.3, the covariance matrix has been maintained fixed.

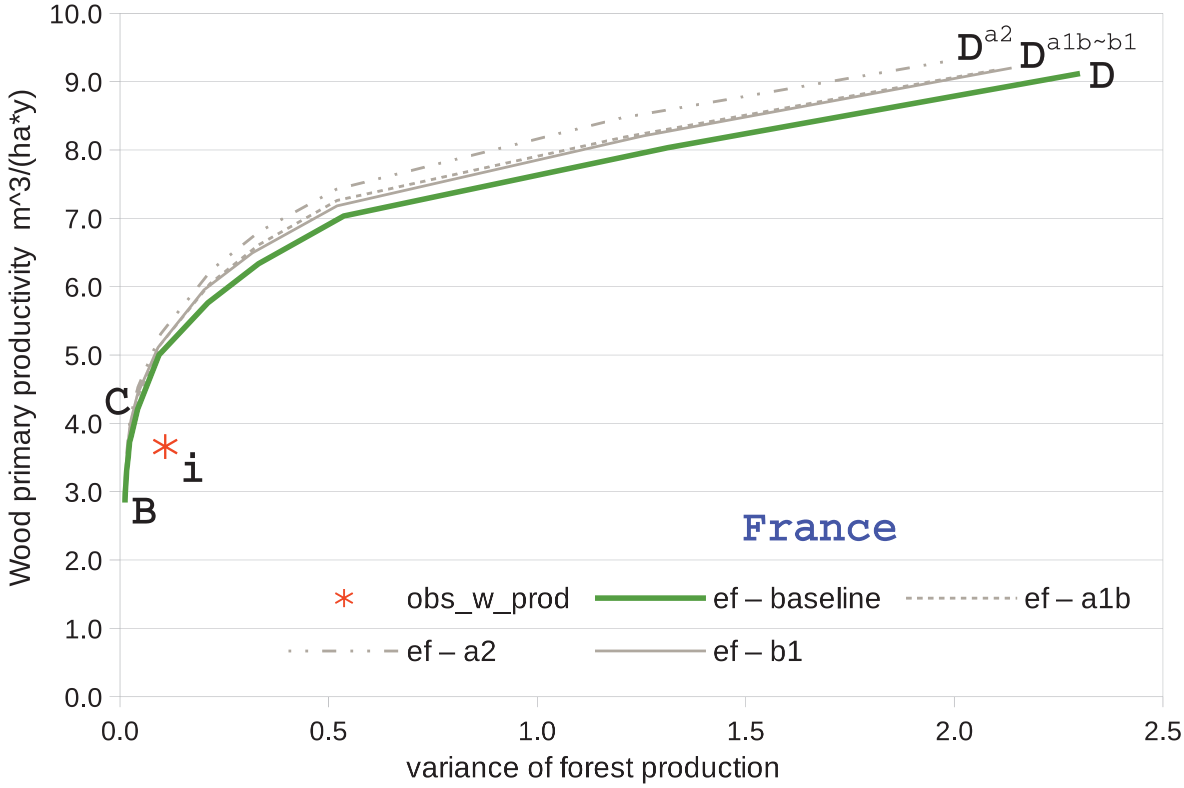
Fig. 4 displays the efficient frontier (EF) calculated from the current productivities (green curve), as well as the allocations relative to the IPCC scenarios (grey curves). We also observe a red star, which represents the actual French allocation. It can be discerned that, at the national level, the empirical forest allocation is close to the efficient frontier. Nevertheless, the high risk aversion that we reveal places the management of French forests at the low parabola coordinates, where the portfolio productivity is not at its highest.

Optimizing the current portfolio for wood production (moving from point `i` to point `C` in Fig. 1), while keeping the same risk aversion, would lead to the respective increases in wood production and sequestration of 3.2 $M\ m^3\ \ y^{-1}$ and 3.2 Mt $\mathrm{CO_2}eq\ \ y^{-1}$ (Table 2). Regarding the economic value, optimizing the wood production, without targeting its economic valorization, would lead to a a value fall of 416 $M€\ y^{-1}$.

Fig. 4 also reports the way the EF would be modified if a specific climate change scenario, defined in Section 2.3, should occur. Given that the covariance matrix is fixed, the EF transposition results from changes in the expected productivity. Thereby, for any given scenario, the respective EF curves coincide at the minimum variance coordinates and diverge as α decreases. Put differently, we find that the climate change scenarios positively affect the frontier, for the productivities increase at any given level of risk. In detail, the a2 scenario performs the best.

For all three objectives (Table 2), the proximity between point `C` and point `B` points out that, contrary to the risk neutral point (`D`), the effects of the intermediate risk aversion and full risk aversion are analogous.

Figure 4: Efficient frontier and actual allocation in France



Efficient frontiers (lines) and actual allocation (red star) for France. Baseline EF is 2009. Climate change scenarios are average values (2020-2100).

Table 2: Wood production, carbon sequestration and economic valorization on different points of the efficient frontier (optimization with respect to the wood production)

Climate change scenarios	a1b	a2	b1	baseline
Wood production ($M\ m^3\ y^{-1}$)				
(currently observed: 55.4 $M\ m^3\ y^{-1}$; 3.52 $Mc\ ha^{-1}\ y^{-1}$)				
Full risk aversion	45.9	47.2	46.6	43.8
Intermediate risk aversion	60.5	62.7	61.1	58.6
Risk neutrality	137.0	139.1	137.4	136.3
Carbon sequestration (Mt CO$_2$$eq\ y^{-1}$)				
(currently observed: 78.8 Mt CO$_2$$eq\ y^{-1}$; 5.01 t CO$_2$$eq\ ha^{-1}\ y^{-1}$)				
Full risk aversion	69.9	72.1	71.1	66.3
Intermediate risk aversion	85.2	88.6	86.6	82.0
Risk neutrality	170.4	172.7	173.0	170.4
Economic valorization (M€ y^{-1})				
(currently observed: 3,789 M€ y^{-1}; 240 € $ha^{-1}\ y^{-1}$)				
Full risk aversion	3,864	3,902	3,887	3,745
Intermediate risk aversion	3,687	3,759	3,684	3,373
Risk neutrality	5,307	5,269	5,415	5,426

Choosing a specific level of risk aversion significantly impacts the compositions in the optimal portfolios. Fig. 5 illustrates the relative species allocation under different assumptions on risk aversion. Three different patterns can be identified.

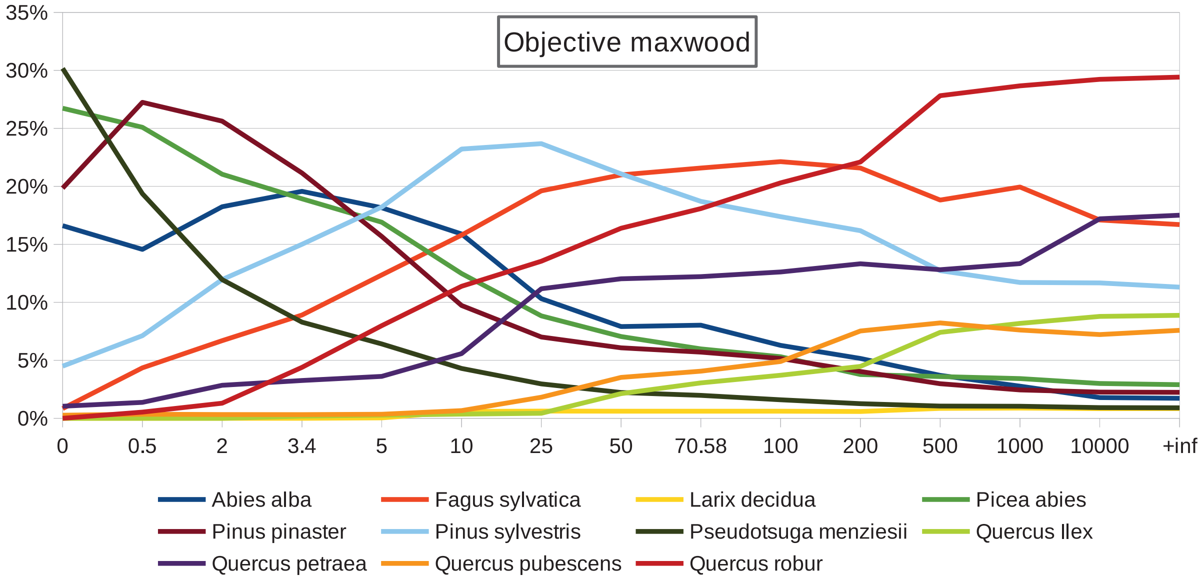
The first one is relative to the species that yield high portfolio productivity and risk: either because they show a high variance or because they are positively correlated with the other species in the portfolio. These species (e.g. *Picea abies*, *Pinus pinaster* or *Pseudotsuga menziesii*) constitute an important part of the portfolio under risk neutrality.

The second pattern includes the species (e.g. *Quercus robur*, *Quercus petrea*, *Quercus pubescent* and *Quercus ilex*) with specular characteristics: they bring stability to the portfolio to the detriment of its productivity. They tend to appear in greater proportions as the risk aversion increases.

The third pattern displays intermediate characteristics and arises under the intermediate risk aversion (e.g. *Pinus sylvastris* and *Fagus sylvatica*). Its coefficients yield the highest portfolio diversification.

One of the critiques being leveled at the M-V model is that it looks at the past variance, such that the variance-covariance matrix is assumed to be constant. While point `B` reflects the most robust portfolio under the risk currently observed, the diversity encountered at the intermediate levels of risk aversion can also ensure an overall stability required for confronting climate change.

Figure 5: Species allocation by risk aversion



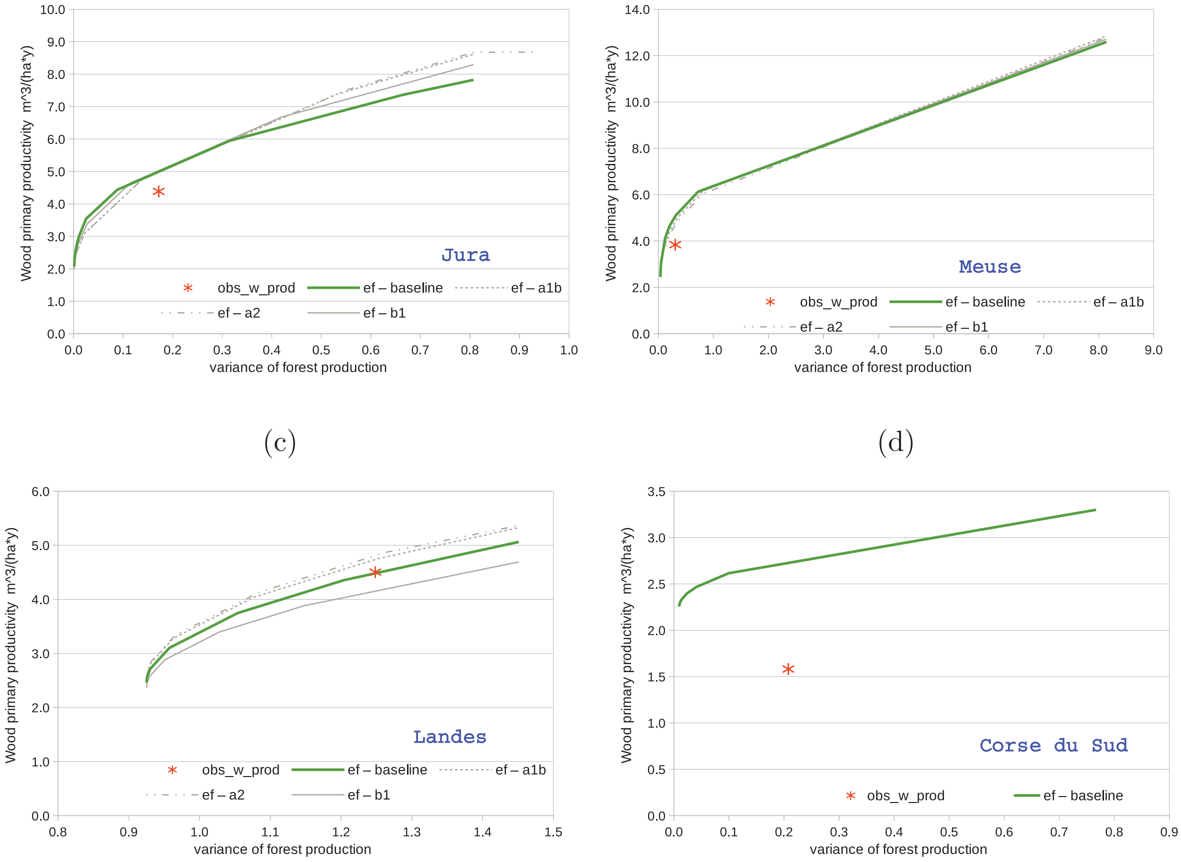
3.1 Regional differences

Working with departmental data allow us to build the efficient frontier at the department level. We note that the majority of departments display allocations similar to the national one, that is, close to the frontier with a preference for low variance at the expense of high productivity. Among others, we can quote `Alpes Maritime`, `Jura` (Fig. 6a), `Meuse` (Fig. 6b).

A few departments turn out to be on the EF, with risk neutrality and high productivity. This is the case of `Landes` (Fig. 6c), `Gironde` and `Haute Savoie`.

At last, some departments are distant from the optimal allocation (`Corse du sud` - Fig. 6d), `Pyrénées-Orientales`).

Figure 6: Efficient frontiers and current allocations in four French departments


(a) (b) (c) (d)

Efficient frontiers (curves) and actual allocation (red star) in four French departments. Baseline EF is 2009. Climate change scenarios are average values (2020-2100).[a]

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[a] Climate change scenarios for `Corse du Sud` are not reported, for they appear as small segments out of the figure.

3.2 Maximizing for which objective?

As we switch objectives in the portfolio optimization, we obtain the performances, at the national level, such as described in Table 3 and the species allocations, such as depicted in Fig. 7.

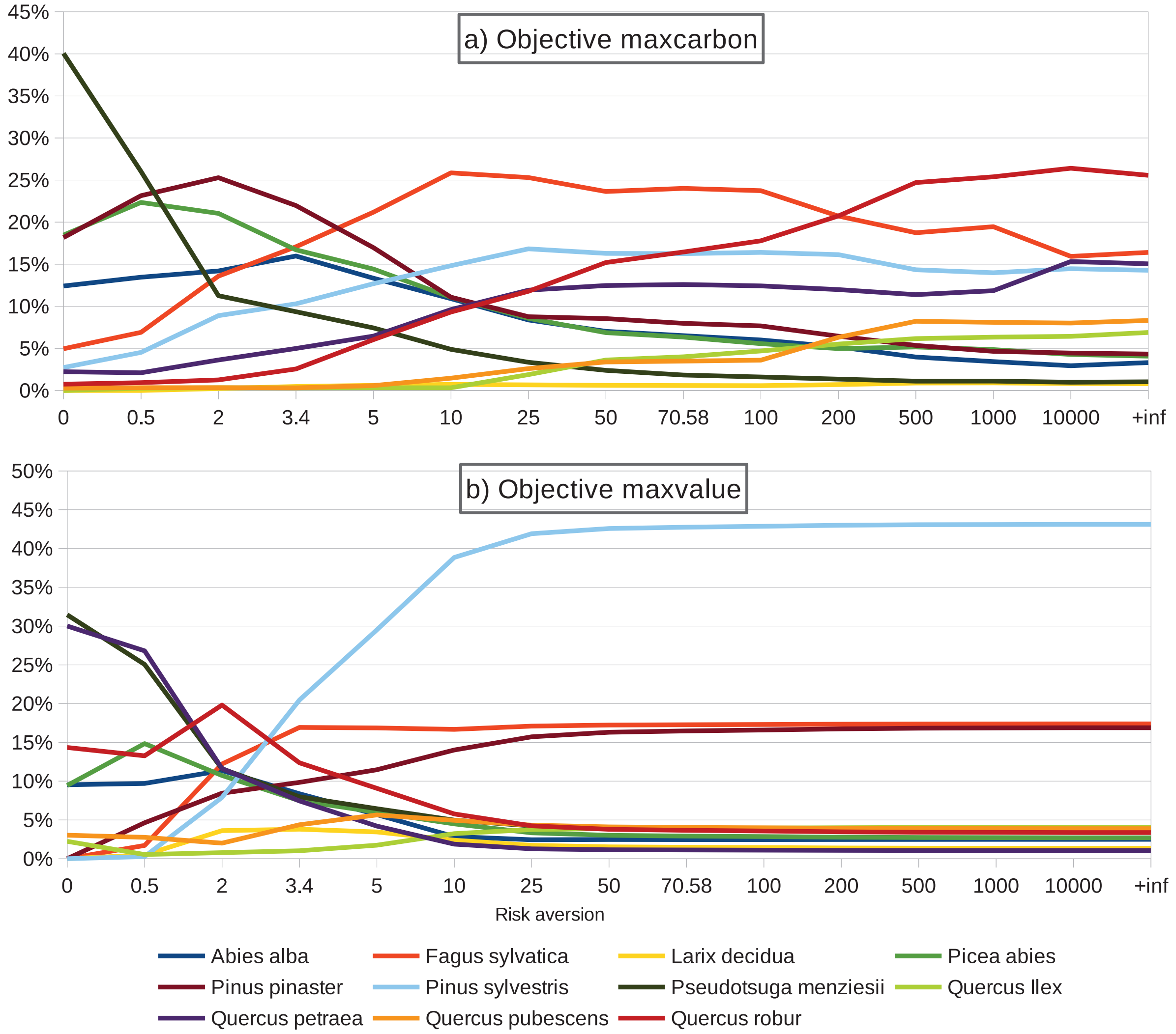
While one would expect the WP to be the highest when the objective is to maximize WP, as with CS and EV, it is proving to be only true for risk neutrality. As the risk aversion increases, risk counts more than the productivity.

Knowing that the species with low prices are also those with low price volatility, ignoring prices leads to more profitable species in the optimal portfolio. This is evident in Fig. 7 (b) where the high risk aversion scenarios favor *Pinus sylvestris*.

When compared to Fig. 5, Fig. 7 shows that the carbon sequestration objective would roughly lead to the same portfolio. As previously stated, we observe differences only under the risk neutrality scenarios, where the high-dense wood species, like *Pseudotsuga menziesii* or *Fagus sylvatica*, supplant the low-dense wood species, like *Pinus sylvestris* and *Abies alba*.

The fact that the empirical allocation is closer to the EF when the optimization covers the physical productivity, rather than the economic valorization, suggests that forest managers react neither on wood prices nor on their volatility. If forest managers were risk averse toward wood prices, the price variances would matter more. However, given the long rotation age in forestry, as well as the impossibility to rapidly reconfigure the distribution of species, the variation of annual prices might not be considered as a risk factor.

Figure 7: Species allocation by risk aversion under three maximization objectives



The empirical performance in carbon sequestration (78.8 Mt $CO_2eq\ y^{-1}$) is very close to what we find at the optimum. Furthermore, the Kyoto protocol stipulates that, through forests, France ought to sequestrate around 66 Mt $CO_2eq\ y^{-1}$ per year up to 2020 (Colin, 2014). This presumes that the Kyoto objectives are either too lax or that the French forests are highly efficient when comes to sequestrating. Thereby, should the France principally aim to produce wood, the Kyoto objective would be equal to the performance achieved in the baseline scenario at the full risk aversion.

In case the objective is to sequestrate carbon, the optimal species distribution is very similar to the one obtained when maximizing wood production.[9]

On the contrary, when the objective is to maximize the economic value of the wood production, the optimal species distribution varies significantly, and this is all the more true as

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[9] Coniferous store less carbon than broad leaves, but other discrepancies across species are limited.

Table 3: Optimization of wood production, carbon sequestration and economic valorization under three maximization objectives

Climate change scenarios	a1b	a2	b1	baseline
Wood production ($M\ m^3\ y^{-1}$)				
(currently observed: 55.4 $M\ m^3\ y^{-1}$; 3.52 $Mc\ ha^{-1}\ y^{-1}$)				
Full risk aversion				
- *obj maxwood*	45.9	47.2	46.6	43.8
- *obj maxcarbon*	49.9	51.2	50.5	47.7
- *obj maxvalue*	62.6	63.4	63.5	62.5
Intermediate risk aversion				
- *obj maxwood*	60.5	62.7	61.1	58.6
- *obj maxcarbon*	62.0	63.9	62.1	59.1
- *obj maxvalue*	62.9	63.9	63.8	62.8
Risk neutral				
- *obj maxwood*	137.0	139.1	137.4	136.3
- *obj maxcarbon*	134.5	136.6	134.5	132.7
- *obj maxvalue*	94.3	94.4	95.0	99.7
Carbon sequestration (Mt $CO_2eq\ y^{-1}$)				
(currently observed: 78.8 Mt $CO_2eq\ y^{-1}$; 5.01 t $CO_2eq\ ha^{-1}\ y^{-1}$)				
Full risk aversion				
- *obj maxwood*	69.9	72.1	71.1	66.3
- *obj maxcarbon*	73.6	75.8	74.6	69.9
- *obj maxvalue*	85.7	87.3	87.2	85.0
Intermediate risk aversion				
- *obj maxwood*	85.2	88.6	86.6	82.0
- *obj maxcarbon*	87.2	90.4	88.2	83.2
- *obj maxvalue*	86.1	87.9	87.7	85.3
Risk neutrality				
- *obj maxwood*	170.4	172.7	173.0	170.4
- *obj maxcarbon*	173.5	175.9	176.4	174.4
- *obj maxvalue*	131.4	131.9	134.6	138.0
Economic valorization ($M€\ y^{-1}$)				
(currently observed: 3,789 $M€\ y^{-1}$; 240 € $ha^{-1}\ y^{-1}$)				
Full risk aversion				
- *obj maxwood*	3,864	3,902	3,887	3,745
- *obj maxcarbon*	3,723	3,761	3,742	3,604
- *obj maxvalue*	1,777	1,797	1,815	1,773
Intermediate risk aversion				
- *obj maxwood*	3,687	3,759	3,684	3,373
- *obj maxcarbon*	3,634	3,721	3,630	3,333
- *obj maxvalue*	1,830	1,854	1,869	1,822
Risk neutrality				
- *obj maxwood*	5,307	5,269	5,415	5,426
- *obj maxcarbon*	5,608	5,560	5,726	5,702
- *obj maxvalue*	7,367	7,401	7,503	7,295

the risk aversion increases. Indeed, forest producers cannot rapidly adapt to price variations.[10] At most, they can either decide to harvest, from the silvicultural optimal portfolio, the species highly-valued by the market at some point of time, or they can decide to postpone the harvesting until the prices increase, but this strategy is restricting in view of the rotation boundaries. The relative unpredictability of prices makes the switch toward economic valorization very risky.

Under the very high risk aversion we have found in France, this would lead to a very low species diversification and more specifically a forest specialization in *Pinus Sylvrestris*. Such configuration may play a role on the effective risk borne by forest owners, since losses can be species specific (such as wood-boring insects and fungi), but lies outside the paper scope. In addition, such an option could be heavily detrimental on carbon sequestration and may radically change the landscape. Indeed, foresters would need to undertake the silvicultural activities on the whole French territory like in the *Landes* department in southwestern France. This is rather unlikely, because many French forests are currently managed through the natural regeneration of existing species. Even though the industrial demand for softwood species is pressing, and the current stocks appear insufficient, in view of their mission statements, both the Ministries of Agriculture and Environment would have to approve such an evolution.

4 Discussion

Using the Markowitz mean-value (M-V) portfolio model, we studied forest planning that allows for considering the trade-off between productivity and risk in an explicit manner. When applied to the French metropolitan territory, our simulations yield a range of possibilities for forest managers to achieve three principal objectives which could be assigned to forest ecosystems.

Among the various services provided by forests, we mainly focused on wood production and carbon sequestration. In case of high level of risk aversion, we found that the empirical performance is close to efficiency. Accordingly, at the current risk aversion level, changing the existing set-up would have no immediate foundation. In particular, given the small differences between the tree species in terms of carbon concentration, maximizing a portfolio for wood production amounts to maximizing it for carbon sequestration. This is our main result.

Knowing that carbon sequestration is almost fully correlated with productivity, when French authorities promote timber production, issued from sustainably managed forests, they turn out to target the fight against climate change. Nevertheless, would the Kyoto objectives, in terms of carbon sequestration, be more constraining in the future, the cursor should be moved to more risk neutrality, because the efficient frontier forecasts enhanced productivity at higher levels of risk. To do so, private mechanisms of risk sharing, such as the insurance contracts, should be implemented, especially in the regions, like the French South-West, where the forest owners are regularly subsidized in case of calamities.

Moreover, we tested the inclusion of wood prices in the economic optimization. Our results reveal that forest owners favor productivity over the profit maximization. Indeed, the empirical allocation does not support the idea that forest managers consider the yearly variation of prices as a risk factor. This could be explained by the long-term rotation of wood species, which goes in the direction opposite to the short-term price volatility, together with the relative flexibility in deciding on the harvesting year. The availability of longer productivity time series would have enabled us to test the perception of prices as a risk factor.

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[10] The same limits on the fixed covariance matrix is happening here.

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A Simulator source code, input data and complete output

The supplementary material is composed of 2 parts:

A.0.1 Simulation program

`loop.7z` includes the program used to produce the simulations reported in this paper (a python script) and the required data. However two input files, productivities and climate change multipliers, are not publicly available as we do not hold their copyright.

A.0.2 Complete output data

`output.7z` contains two OpenDocument spreadsheet files. `spAllocation.ods` contains, for each run simulation, the portfolio's weigth for each species. `depPerformances.ods` includes instead the consequent "performances" of such optimal portfolios for the dimensions analysed in the text.